Investment Performance Improvement Utilizing

Automated Polymorphic Momentum

Abstract

While it's now generally accepted by academics[1,2,3] and industry professionals[4,5] alike that momentum is the premier market anomaly, there is typically neither rationale nor defense for the six- or 12-month look back periods[6,7] commonly used in academic studies measuring momentum. Notably, Information Theory[t1] and Detection Theory[t2] dictate that the probability of making an excellent investment decision for a subsequent period of time will be directly proportional to the signal-to-noise ratio[t3] of the employed momentum indicator signal. This paper will (a) examine how the cross-disciplinary sciences of Matched Filter Theory[t4] and differential signal processing[t5] can be used to measurably improve the signal-to-noise ratio, (b) show that different sets of equities require different momentum filter functions, and (c) demonstrate that a momentum filter with both adaptive shape and duration has substantial value.

Introduction

Momentum is loosely defined as the tendency of a moving object to continue moving. However, the term is used very differently by momentum traders and trend followers. Momentum traders care deeply about the underlying story, such as improved earnings or rocketing sales, and define momentum as (trading volume) times (price change) in much the same way as physicists define momentum as mass times velocity. However, trend followers generally don't care why the price has momentum; they simply believe that the trend is their friend. In that context, Automated Polymorphic Momentum is simply about extracting trend signals from noisy market data in a manner that is most predictive of next month's performance.

In the analysis that follows, the term "strategy" refers to a set of 12 candidate funds that are evaluated by a trend measurement algorithm at the end of each month to determine which one – and only one – of the 12 candidates will be owned during the next month. The set of 12 candidate funds for each strategy is selected randomly from one of the three fund sets: Fidelity General, Fidelity Sectors, and ETFs Pre-2007, as detailed in Appendices A, B, and C, respectively. Large sets of random strategies are created and evaluated using the Strategy Evaluation tool of Appendix D, which employs the same thoroughly-validated strategy algorithm engine embedded in both the SectorSurfer and

AlphaDroid automated investment advice services (first debuting online in 2010). High quality market data is provided by FastTrack.

While momentum in market data has been well studied and generally accepted as the premier market anomaly,[1,2,3] its researchers often simply default to using either a six- or 12-month look back period[6,7] without equally considering the methodology of how momentum should be measured. This paper will demonstrate the following:

(a) the SMA-125 (simple moving average of 125 market days, or 6 months) and the SMA-250 (12 months) are not optimum measures of momentum,

(b) the set of candidate funds (which can change over time) significantly affects the strategy group dynamics and how best to measure relative momentum, and

(c) the decision of whether to be in or out of the market is different from deciding which fund to own and leads to separate bull and bear market solutions.

Fortunately, the cross-disciplinary science of Matched Filter Theory[t4] and differential signal processing[t5] can be employed to measurably improve the probability of making excellent investment decisions. Furthermore, momentum filters, adjustable in shape and duration (polymorphic), can be developed to automatically adapt to the candidate set of funds. The first step of this process is to assess the scope of the problem.

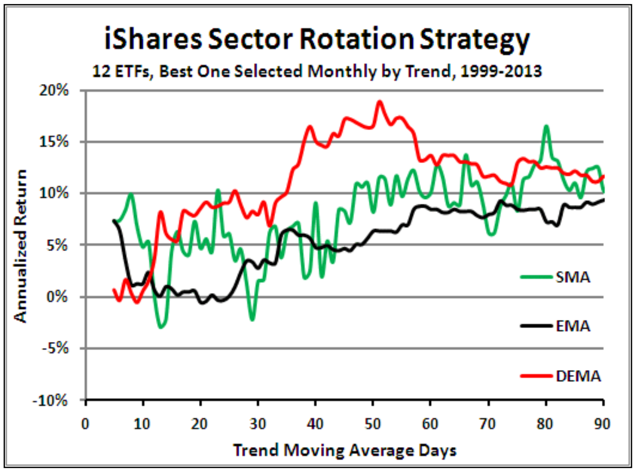
Which Trend Is Your Friend?

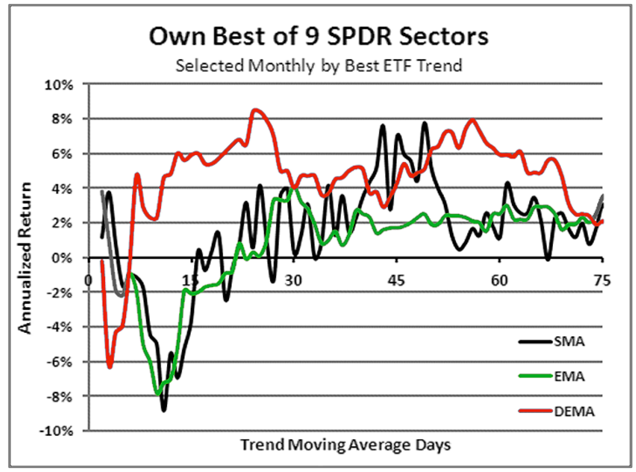
While a fund's momentum is simply a measure of its recent trend, the numerous trend measurement algorithms (such as SMA, EMA, DEMA, RSI, MACD, etc.) and wide range of trend time periods (such as week, month, quarter, year, etc.) make it clear that a deterministic selection method is required. The scope of the problem is illustrated in Figure 1, which contrasts the performance of 30 strategies, each composed of a set of 12 randomly selected Fidelity Sectors funds (Appendix B). Each of its seven colored columns utilizes the momentum algorithm specified by its column header to determine the annualized return for each strategy. The time constant, in days, follows the standard SMA, EMA, and DEMA abbreviations for the "simple moving average," "exponential moving average," and "double exponential moving average," respectively. The SMA-125 and SMA-250 correspond to the six-month and 12-month look back periods, respectively. The highest performing strategies (funds plus algorithm) have green backgrounds, while the worst performing have red backgrounds. Although the table is sorted by the SMA-250 column's performance, the colors in other columns still appear quite randomly distributed. Notably the SMA-250 beats all other algorithms for only six of 30 strategies. That is not the character one might expect of the SMA-250, given its prominence in momentum research papers. On the other hand, there is no obvious better choice – other than developing an adaptive solution.

First 30 of 200 Strategies of 12 Random Fidelity Sector Funds													Annualized Return for 12/31/2005 to 12/31/2015						
Strat#	Fnd.1	Fnd.2	Fnd.3	Fnd.4	Fnd.5	Fnd.6	Fnd.7	Fnd.8	Fnd.9	Fnd.10	Fnd.11	Fnd.12	SMA-63	SMA-125	SMA-250	EMA-63	EMA-125	DEMA-21	DEMA-125
1	FSHOX	FSDAX	FSPHX	FSLEX	FCYIX	FSCSX	FSTCX	FPHAX	FSHCX	FBIOX	FSAIX	FSCGX	6.9	7.18	14.34	9.2	6.59	7.11	7.24
2	FSCPX	FSPTX	FSCGX	FSLBX	FSRPX	FSHCX	FSAVX	FSTCX	FBMPX	FSDPX	FSENX	FBIOX	11.18	12.19	14.29	14.71	18.82	8.41	15.14
3	FSHCX	FSPHX	FSAIX	FSCHX	FSPCX	FPHAX	FSRBX	FCYIX	FSDAX	FNARX	FBMPX	FSPTX	8.12	6.46	14.02	4.33	7.37	8.04	7.32
4	FSAVX	FSPHX	FDCPX	FSCHX	FBSOX	FBIOX	FSMEX	FSLEX	FDFAX	FSCSX	FIDSX	FSAIX	10.42	7.98	13.68	14.02	13.14	9.82	14.85
5	FSRBX	FSNGX	FSDCX	FSPHX	FSAIX	FSLEX	FSRPX	FSDPX	FSVLX	FBSOX	FCYIX	FSPCX	4.2	6.66	13.43	0.96	1.99	1.24	7.07
6	FBIOX	FSRFX	FWRLX	FDFAX	FSRBX	FSCSX	FSUTX	FSHOX	FSPTX	FSAVX	FSAIX	FSLEX	7.21	6.25	12.42	12.6	12.26	9.21	10.52
7	FIDSX	FBMPX	FSCGX	FSTCX	FDCPX	FNARX	FDLSX	FSRPX	FSVLX	FSENX	FSAIX	FWRLX	8.02	7.79	12.15	8.71	7.14	5.74	8.38
8	FIDSX	FSMEX	FWRLX	FSPHX	FSVLX	FSELX	FSCGX	FDCPX	FSCHX	FSRFX	FBMPX	FSESX	3.92	4.55	11.12	5.68	7.34	9.06	7.62
9	FSPHX	FBIOX	FSMEX	FCYIX	FSDAX	FSCGX	FSPTX	FSCHX	FSELX	FSDPX	FSAVX	FSESX	11.28	20.49	10.74	15.17	21.56	13.13	9.53
10	FSVLX	FSESX	FSPHX	FSPTX	FDCPX	FSLBX	FSCPX	FSHOX	FDFAX	FBSOX	FSTCX	FSPCX	7.96	5.47	10.24	7.51	7.37	8.12	4.98
11	FSDPX	FSENX	FBIOX	FPHAX	FNARX	FSDCX	FSRBX	FSTCX	FBSOX	FSPTX	FSHCX	FSCHX	11.45	9.18	9.99	12.61	15.72	10.92	12.38
12	FBIOX	FSAIX	FDCPX	FCYIX	FSHCX	FSRPX	FSENX	FSCGX	FDLSX	FSDCX	FSPCX	FSCHX	8.69	6.72	9.87	8.34	9.74	5.03	11.32
13	FSMEX	FSRBX	FDLSX	FSNGX	FSHOX	FPHAX	FNARX	FSVLX	FSDPX	FSENX	FBIOX	FBMPX	11.68	14.28	9.47	4.9	12.25	10.78	11.45
14	FSDAX	FBIOX	FSCGX	FSCHX	FWRLX	FSDCX	FBSOX	FSENX	FSLEX	FSPHX	FSHCX	FSRPX	10.78	10.95	8.66	12.01	13.63	7.9	12.99
15	FBSOX	FSRFX	FSHOX	FBIOX	FDFAX	FSCSX	FSRPX	FSHCX	FSAVX	FSDCX	FSENX	FSLBX	9.34	12.91	8.65	16.23	16.98	8.33	15.14
16	FSRFX	FDCPX	FSPTX	FSUTX	FSPHX	FSDPX	FSRBX	FSMEX	FSDCX	FPHAX	FDLSX	FNARX	13.05	4.41	8.57	9.62	6.64	11.29	6.21
17	FSESX	FBIOX	FSPCX	FSRBX	FSDAX	FSPHX	FNARX	FIDSX	FSELX	FSAVX	FSCHX	FSCSX	11.35	17.31	8.11	13.5	18.94	11.59	9.3
18	FBSOX	FDLSX	FSNGX	FIDSX	FSLBX	FSCPX	FSDCX	FSELX	FDCPX	FSRPX	FDFAX	FWRLX	8.36	5.44	8.08	5.98	8.88	6.64	1.23
19	FCYIX	FSHCX	FSAIX	FSVLX	FSDAX	FSPHX	FSCPX	FSMEX	FSTCX	FDLSX	FSHOX	FSLBX	6.07	9.43	7.86	5.21	6	7.42	7.18
20	FSTCX	FSMEX	FSRPX	FSCHX	FSVLX	FSCGX	FSUTX	FWRLX	FSAIX	FSDPX	FSRFX	FSHOX	9.4	11.41	7.68	14.19	10.95	9.5	5.43
21	FIDSX	FPHAX	FDCPX	FSMEX	FSCGX	FSAVX	FSRPX	FSPHX	FSLEX	FSCSX	FSRBX	FSHCX	8.87	8.54	7.65	8.95	7.37	12.23	7.7
22	FSDCX	FNARX	FDLSX	FSPTX	FSRPX	FSCHX	FSAVX	FDCPX	FSAIX	FSCPX	FSESX	FSHOX	8.2	7.14	7.54	11.7	8.57	3.47	7.17
23	FSPHX	FSCPX	FSAVX	FBMPX	FDCPX	FSHOX	FSAIX	FBSOX	FSNGX	FDLSX	FSCHX	FSDPX	6.91	9.31	6.83	7.91	6.7	3.76	7.74
24	FSESX	FSCHX	FSPHX	FDLSX	FWRLX	FDCPX	FBSOX	FIDSX	FNARX	FSMEX	FSDCX	FSELX	11.27	6.51	6.8	11.04	9.87	11.51	2.82
25	FNARX	FSCGX	FIDSX	FWRLX	FSHCX	FSLEX	FSLBX	FSCHX	FSAVX	FBMPX	FDFAX	FSRBX	11.15	4.51	5.8	10.1	4.18	8.28	6.73
26	FSCHX	FSPHX	FSVLX	FSPCX	FSNGX	FDFAX	FSESX	FSDPX	FNARX	FDLSX	FSUTX	FSCGX	8.2	5.86	5.24	11.61	6.22	9.09	4.56
27	FSAIX	FDFAX	FSCHX	FSAVX	FSDPX	FPHAX	FSNGX	FCYIX	FSCPX	FSHOX	FSDAX	FBMPX	9.13	5.49	5.17	1.11	5.04	7.08	6.56
28	FNARX	FSVLX	FDLSX	FSAVX	FCYIX	FSELX	FSPCX	FSHCX	FSCPX	FSCGX	FSLEX	FSESX	6.02	12.04	4.59	11.24	15.66	11.26	2.76
29	FSDPX	FBMPX	FSAVX	FSRBX	FIDSX	FDCPX	FSDAX	FSCGX	FSELX	FCYIX	FSVLX	FSCHX	10.32	3.67	4.4	9.03	9.55	10.11	13.73
30	FSLBX	FSLEX	FSAVX	FBMPX	FIDSX	FBSOX	FSDAX	FSELX	FNARX	FSVLX	FWRLX	FSAIX	13.41	7.65	4.01	18.5	9.05	11.49	8.17

Figure 1. A performance heat map of 30 random Fidelity sector strategies using seven algorithms sorted by the SMA-250 algorithm performance.

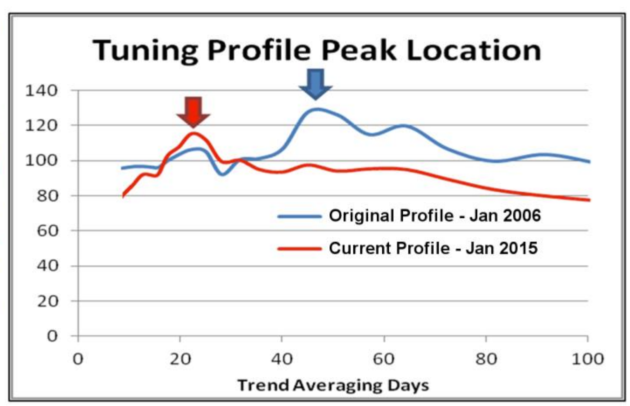
A more detailed understanding of this problem is illustrated by the difference in strategy performance between Figures 2a and 2b. The strategy of Figure 2a is composed of a set of 12 iShares sector ETFs, whereas the strategy of Figure 2b is composed of a similar, but not identical, set of 12 SPDR sector ETFs. Strategy performance was evaluated using the SMA, EMA, and DEMA algorithms with time constants spanning 5 to 75 days. While the DEMA algorithm (red) performed better in both, the performance peak for the iShares strategy occurred at about 25 days, whereas the performance peak for the SPDR strategy occurred at about 50 days.





Figures 2a and **2b**. Strategy performance plotted against the trend time constant for two different sets of ETFs using three different trend algorithms.

Further complicating matters are strategies, such as the one in Figure 3, where some funds first participate much later than others. In this strategy, Fidelity FMAGX, FLCSX, and FMCSX are initially the only funds participating. In mid 2006, the set of candidate funds begins to include sector ETFs ITB, XPH, and XSD. By 2008, XRT starts participating, and finally BIB joins the pack in 2010. In other words, if a sports team morphs from a stodgy running team to an exciting passing team, fans expect the speed



Figures 3a and **3b**. Tuning profile changes when new funds start participating.

of the game to change and different players to be handling the ball. Figure 3b shows that the January 2006 tuning profile peak for the original three “stodgy” (well diversified) Fidelity funds was approximately 45 days (blue), whereas the addition of the more volatile and “exciting” sector funds moved the tuning profile peak to about 22 days (red). The team players set the speed of the game.

A “one size fits all” approach to momentum ranking is thus quite far from optimum, as one might expect given the very different characters of diversified funds, bond funds, sector funds, country funds and commodity funds. Simply stated, the general answer to the question - Which trend is your friend? - is the one that most faithfully predicts which of the funds will produce better returns next month.

Thus, the full scope of the problem requires a solution that includes the design and implementation of an adaptive momentum filter in addition to the application of the principles of Matched Filter Theory and differential signal processing. It’s all about selectively reducing the noise to better reveal the signal. The subsequent sections of this paper will focus on developing a composite solution, starting with the theory and application of differential signal processing.

Differential Signal Processing

Differential signal processing is a method for removing noise that is common to a pair of signals (known as common mode noise) by subtracting the value of one from the other. This can be appreciated in Figure 4, where some funds have more similarities in daily price movements than others. FDCPX, FSHCX, FSVLX, FBMPX, and FSTCX are Fidelity sector funds, whose daily price movements are much more similar to one another than to either the bond fund (FBIDX) or the gold miner fund (FSAGX). Consequently, an overall reduction in system noise (and improvement in the signal-to-noise ratio) could be expected in a strategy utilizing a differential comparison between the sector funds. However, since the bond fund (FBIDX) is relatively flat in comparison to any of the sector funds, the full complement of the sector fund noise would remain if differentially compared to the bond fund. The gold miner fund (FSAGX) actually adds

Figure 4. Common mode noise.

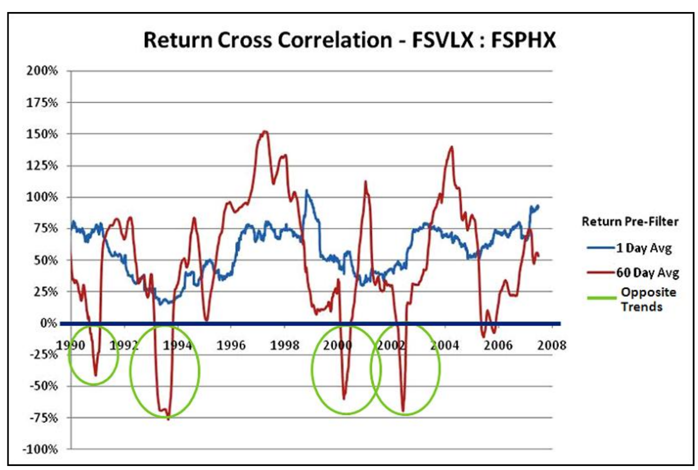


Figure 5. 1-day vs. 60-day fund correlation.

noise in a differential comparison and thus has a well-deserved reputation as a strategy spoiler: higher remnant noise reduces the probability of a good decision.

Figure 5 illustrates the significant difference between a daily correlation and a 60-day average correlation between the Fidelity sector funds FSVLX (Consumer Finance) and FSPHX (Healthcare). While the daily correlation (blue) suggests strong similar behavior between the sectors, when a 60-day moving average filter is applied to the data to remove short-term common mode noise, something wholly different emerges. The negative correlation spikes (green circles) are periods when longer-term fund trends are opposite one another – one fund is increasing while the other is decreasing. Removing short-term common mode noise improves the signal-to-noise ratio, better reveals the trend signal, and improves the probability of a making a better decision.

To appreciate the value of differential signal processing in a trading environment, it must be contrasted against "solo signal processing," which means making a decision by looking only at a single fund. In Figure 6a, FSHCX (Fidelity Medical Delivery) is plotted in green along with a money market fund in red. The investment rule is that FSHCX will be owned whenever the trend for its daily return is higher than that of the money market fund: The money market fund is owned otherwise. There are 10 little yellow

Figure 6a. Solo signal processing.

Figure 6b. Differential signal processing.

bars on the bottom chart, each indicating a trade from one to the other. The results of these trades are shown on the top chart by the yellow line. Clearly, performance is poor, primarily because it is too often the victim of whip-saw losses. However, when FSHCX and FSELX (Fidelity Electronics) are played against one another (6b) to eliminate their common mode noise from the decision process, the results are completely different. Now there are only three trades. Now whip-saw losses have disappeared and the trade decision results are comparatively spectacular. Investing is not a solo contest, it's a horse race – change horses to stay on the fastest horse.

The ramifications of differential signal processing critically imply that the decision to go to cash during a bear market should <u>not</u> be made by 12 instances of solo signal processing similar to that of Figure 6a. However, because it's a comparatively simpler problem, it's an excellent candidate for optimization utilizing Matched Filter Theory.

Matched Filter Theory

A matched filter[t4] is the optimal linear filter for maximizing the signal-to-noise ratio in the presence of additive stochastic noise. In simple terms, this means the best filter shape matches the signal shape. Consider, for example, the radio spectrum signal (yellow) of Figure 7a in the frequency domain. A matched filter might be shaped (as shown in red) to match the spectrum of the desired signal to maximally reduce extraneous sideband noise. However, in the time domain (as required for time series market data) a matched filter (Figure 7b) is a bit more complex to describe. Consider an event (purple) and the time domain signal (red) that leads up to it: If there are many such example events, the pre-event correlation signal (green) leading up to the event can be calculated. In the time domain, a matched filter has an impulse response[t7] (black) that is the mirror image shape of the correlation signal. An impulse response is the filter's extended reaction to a single short event – like striking a bell.

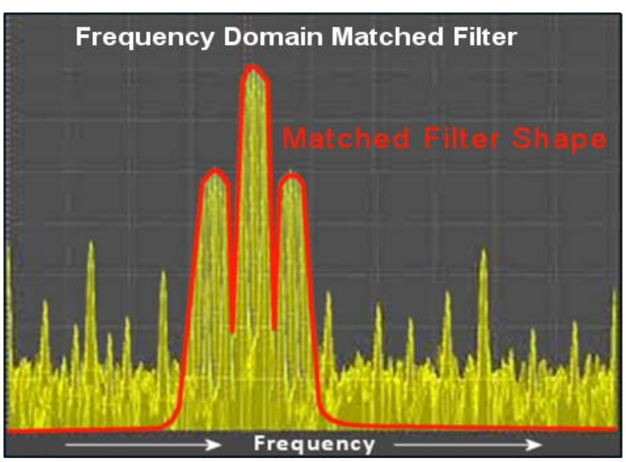


Figure 7a. Frequency domain filter.

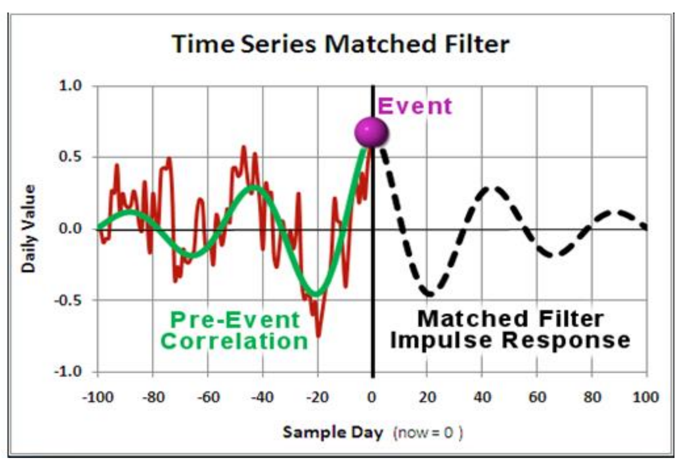


Figure 7b. Time domain matched filter.

The problem to be solved is determining whether or not to move to the safety of cash next month – a bear market strategy. Thus, the event to be predicted is next month's market performance. The S&P 500 Index is used as a proxy for overall market health because negative returns are, of course, indicative of poor market health. The solution requires finding the pre-event correlation between next month's return and the returns from each prior day in order to determine the impulse response of the optimum trend extraction filter. The chart of Figure 8 is that correlation. The horizontal axis is the number of days preceding the month, and the vertical axis is the correlation of the prior data to the subsequent month's return. It has near zero correlation for days immediately preceding the month, and grows to a peak a few months back in

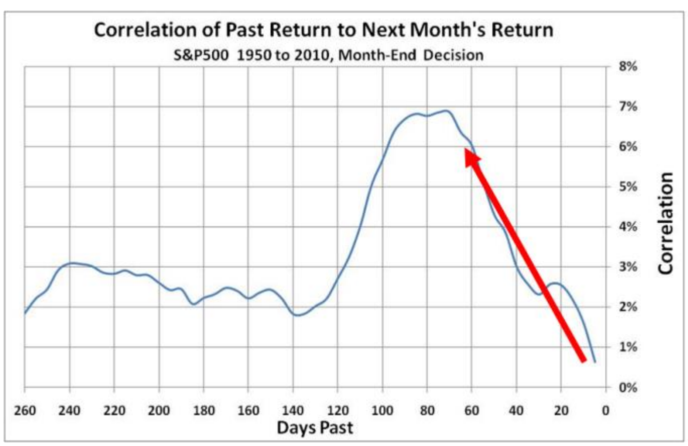


Figure 8. Correlation to next month's return.

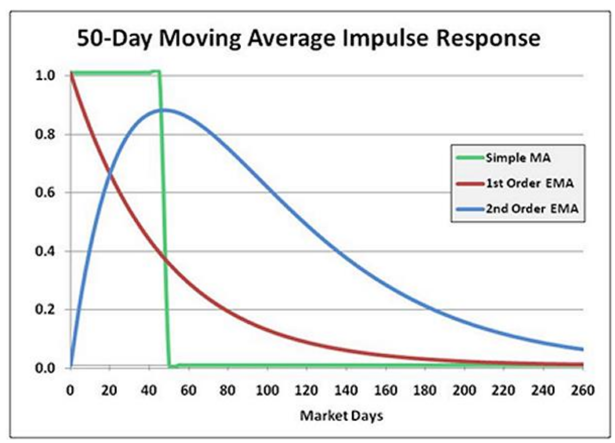


Figure 9. Filter impulse responses.

time. The ideal trend extraction filter would have an impulse response that is the mirror image of the correlation function. However, simplifying the task with an easy to implement approximation may be satisfactory. Figure 9 illustrates the impulse response of three fairly well known filters that are easily evaluated. In green is a 50-day SMA filter. Its impulse response is flat for 50 days and then goes to zero – meaning it will equally weight all data for the past 50 days. In red is a 50-day EMA – quite popular in market data analysis – declining exponentially to its 1/e point at 50 days. Finally, in blue is a 50-day second order EMA (or DEMA filter) with a humped impulse response that most closely resembles the mirror image of the correlation data.

When the correlation data is run through these filters with different time constants, Figure 10 confirms that the DEMA trend filter (blue) outperforms the others. The bear market strategy feature (StormGuard) for both SectorSurfer and AlphaDroid uses the 50-day DEMA to decide when it is time to move to cash. The plots of Figure 11 show the return of multiple operating SectorSurfer Strategies versus different time constants

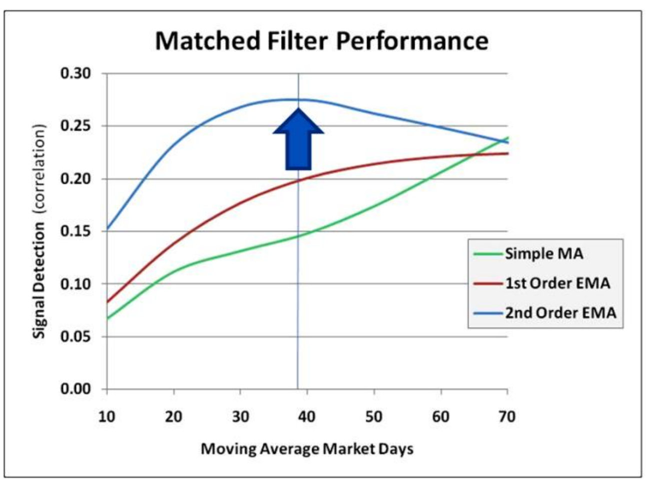


Figure 10. Matched filter performance.

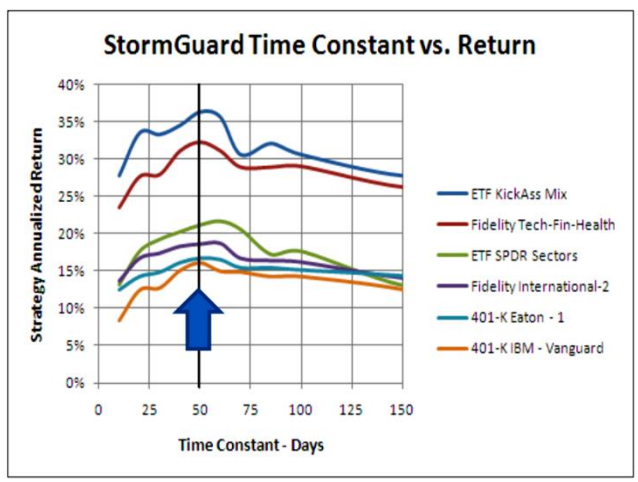


Figure 11. Actual performance.

for the filter. Their performance peak at about 50 days is not an accident. Although the natural human instinct is to pull the trigger to move to cash more quickly, these results suggest there is an important balance to consider: Reacting too quickly to a market event can generate whipsaw losses when sharp drops often snap back, but reacting too slowly only serves to accelerate losses if it is a major market collapse.

The market pull back in the summer of 2010 (red circle Figures 12) was StormGuard's first "real-time" test since its creation. The StormGuard Indicator chart of Figure 13 shows StormGuard came close to triggering a move to CASH, but did not. In the larger

Figure 12. The 2010 and 2011 pullbacks.

Figure 13. StormGuard: DEMA-50d.

perspective, when compared to serious market downturns, StormGuard made the appropriate decision. However, one year later in 2011, the US Debt Downgrade and a threat of collapse of Greek debt scared the market a bit more (blue circle, Figure 12) , and for just a short time, StormGuard did trigger a move to cash.

True Sector Rotation

The proficient utilization of both differential signal processing and Matched Filter Theory leads to a strategy model called True Sector Rotation, as diagramed in Figure 14 for the simplistic case of two funds. Market data is first processed by a Matched Filter to extract the optimum trend signal, which is then compared against the trend signals of other candidates to determine which one – and only one – of them to own now. While other sector rotation strategies engage in some degree of over/under weighting of each sector fund, they never commit all resources solely to the trend leader.

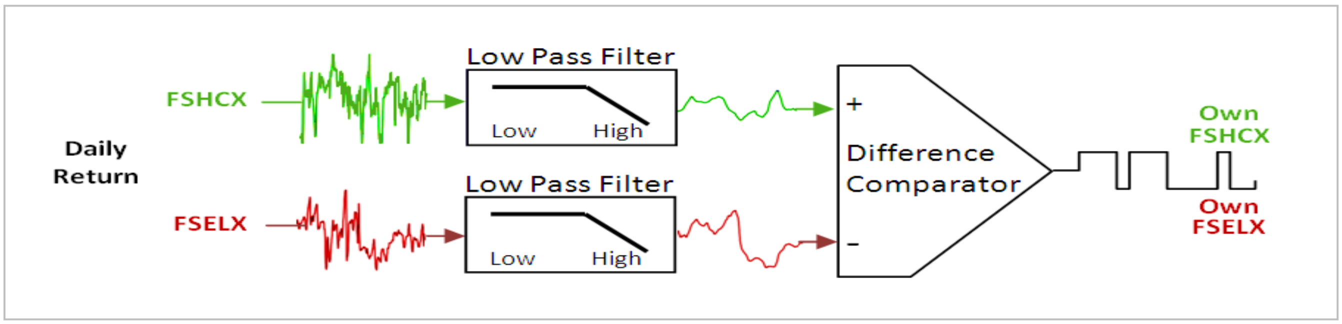


Figure 14. True Sector Rotation model. Own only the trend leader in a set of funds.

The primary argument made for not committing all resources solely to the trend leader generally involves concern that reduced diversification will produce higher risk. However, just the opposite is true, as illustrated in the True Sector Rotation "ETF SPDR Sectors" strategy of Figure 15. Over the 20.7 year period of the strategy, the Sharpe ratio of the S&P 500 is 0.34, while the Sharpe ratio of the strategy is 1.18. There are three factors that cause this to occur: (1) single company risk is the most significant reason for diversification, but it is already well diversified within sector funds; (2) short-term volatility indeed may be higher (due to lower comparative diversification), but True Sector Rotation provides the benefits of “serial diversification” (owning many funds over time, but only one at any given time), which inherently reduces medium-term risk by avoiding poorly performing funds; and (c) the strategy includes the benefit of StormGuard (red circles), which further reduces risk by moving to cash and avoiding market crash losses. The yellow dots on the horizontal axis mark the algorithmic process of forward-walk progressive-tuning (FWPT) to improve overall confidence.

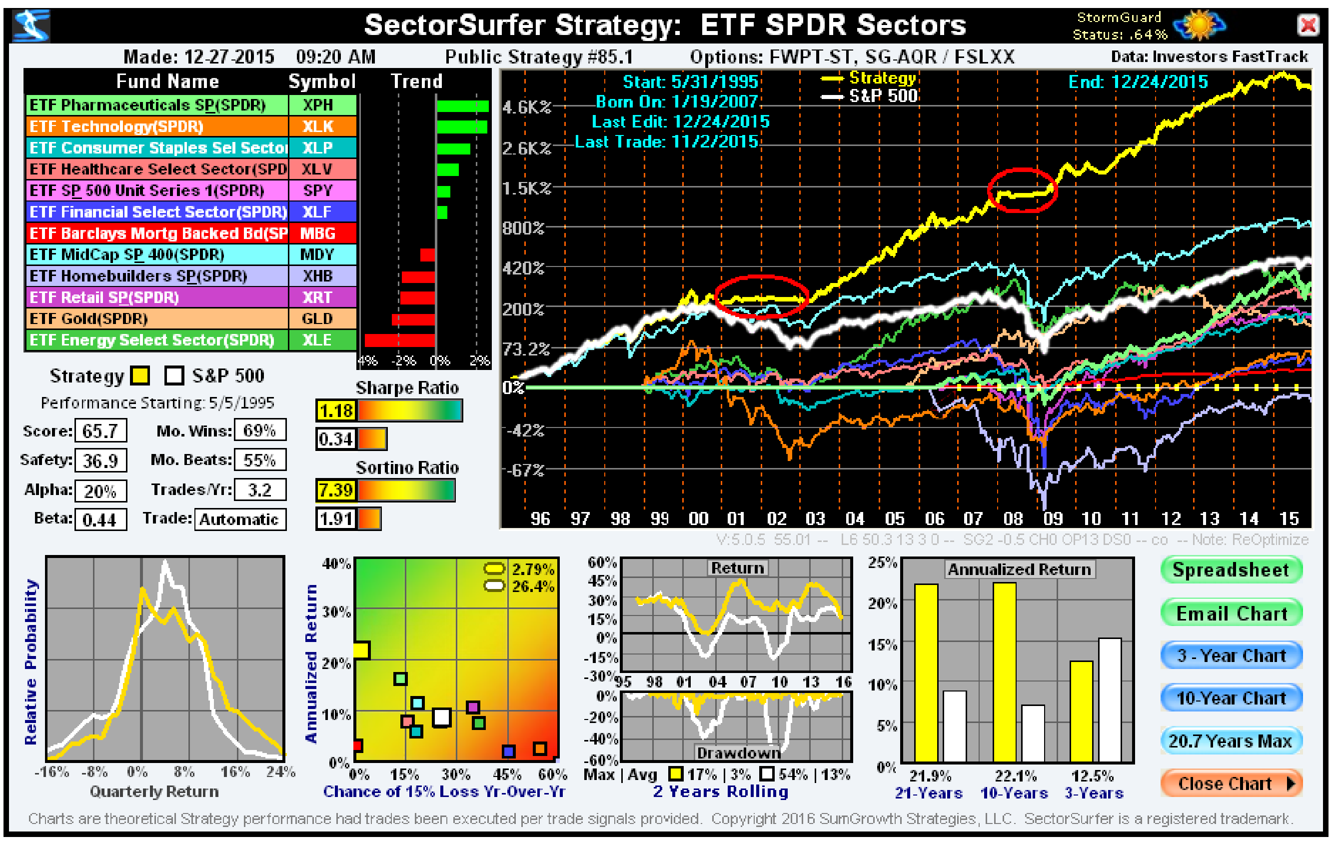


Figure 15. A True Sector Rotation strategy illustrating StormGuard functionality.

Momentum Algorithm Performance

As aforementioned, Figures 1, 2, and 3 demonstrate that the best trend extraction filter is dependent on the set of candidate funds that can compete for selection as the trend leader. To further examine this phenomenon, 58,800 random strategies were evaluated from 12/31/2005 through 12/31/2015 in the below categories, producing the performance tables in Figure 16. The categories are as follows:

- Three fund sets: Fidelity General, and Fidelity Sectors and ETFs Pre-2007
- Three bear market strategies: none, Dual Momentum, StormGuard
- Four algorithms: SMA, EMA, DEMA and TEMA (triple exponential moving average)
- Four time constants: 21 days, 63 days, 125 days and 250 days

The Strategy Evaluation Tool of Appendix D was used to perform the calculations. The first column, entitled “Monkey,” indicates the average results that would be achieved by a large number of monkeys randomly selecting funds to own each month. All momentum strategies select the trend leader at the end of each month and hold it for one month. While Single Momentum strategies have no bear market strategy, Dual Momentum strategies additionally move to cash if the trends of all candidate funds are negative, and StormGuard Momentum strategies will also move to cash whenever the StormGuard indicator so dictates. While some combinations of algorithm and time constant can easily be dismissed, clearly no single combination always performs well.

Figure 16. Strategy performance 12/31/2005 to 12/31/2015 for Single, Dual, and StormGuard Momentum algorithms for different (a) fund sets, (b) trend algorithms, and (c) trend measurement time constants.

		Simple Moving Average				Exponential Moving Avg			Double EMA			Triple EMA		
Single Momentum:	Monkey	SMA-21	SMA-63	SMA-125	SMA-250	EMA-21	EMA-63	EMA-125	DEMA-21	DEMA-63	DEMA-125	TEMA-21	TEMA-63	TEMA-125
Fidelity General:	6.9%	7.5%	9.2%	6.7%	7.8%	7.4%	7.5%	7.3%	7.7%	7.0%	6.4%	7.6%	5.7%	4.8%
Fidelity Sectors:	8.2%	5.0%	9.5%	7.7%	7.7%	3.4%	9.3%	10.3%	9.6%	6.4%	5.7%	8.2%	5.8%	8.2%
ETFs Pre-2007:	6.1%	3.3%	6.7%	6.6%	7.6%	2.3%	6.1%	7.5%	6.4%	7.5%	5.8%	8.3%	5.1%	5.0%
Dual Momentum:	Monkey	SMA-21	SMA-63	SMA-125	SMA-250	EMA-21	EMA-63	EMA-125	DEMA-21	DEMA-63	DEMA-125	TEMA-21	TEMA-63	TEMA-125
Fidelity General:	6.9%	8.2%	12.2%	8.2%	8.5%	8.3%	10.8%	10.5%	11.4%	8.6%	5.0%	11.4%	8.6%	5.0%
Fidelity Sectors:	8.2%	4.2%	11.4%	8.3%	8.5%	2.5%	8.8%	12.3%	10.0%	8.8%	5.8%	10.2%	5.0%	5.1%
ETFs Pre-2007:	6.1%	3.6%	7.3%	7.7%	7.9%	2.0%	6.8%	7.9%	6.3%	8.2%	5.2%	9.0%	4.8%	3.8%
StormGuard Momentum:	Monkey	SMA-21	SMA-63	SMA-125	SMA-250	EMA-21	EMA-63	EMA-125	DEMA-21	DEMA-63	DEMA-125	TEMA-21	TEMA-63	TEMA-125
Fidelity General:	6.9%	12.2%	13.2%	11.8%	11.7%	12.3%	12.2%	12.5%	12.5%	11.8%	11.7%	12.1%	10.6%	11.1%
Fidelity Sectors:	8.2%	9.3%	13.1%	13.5%	14.1%	7.3%	13.6%	15.4%	14.5%	13.3%	14.7%	14.3%	13.7%	15.1%
ETFs Pre-2007:	6.1%	6.8%	10.1%	11.7%	12.6%	6.5%	10.3%	12.0%	10.0%	12.6%	12.8%	12.4%	11.9%	11.2%

Notes: **(58,800 total strategies evaluated for this matrix)**

1. Fidelity General: 200 strategies evaluated – 12 random funds selected from Appendix A list of 53 well diversified funds.

2. Fidelity Sectors: 200 strategies evaluated – 12 random funds selected from Appendix B list of 37 sector funds.

3. ETFs Pre-2007: 1,000 strategies evaluated – 12 random ETFs selected from Appendix C list of 325 ETFs of all kinds.

4. Fidelity General and Fidelity Sectors have no money market funds, no bond funds, and no treasury funds among them.

5. ETFs Pre-2007 included bond funds and treasury funds of all types.

Comparative performance of the Dual Momentum and StormGuard Momentum strategies confirm the value of using Matched Filter Theory to determine the optimum solution for moving to the safety of cash. While performance of StormGuard Momentum strategies is uniformly about 5.5% higher than for Single Momentum strategies, the Dual Momentum strategies uniformly fail to do as well in their best categories. Notably, Dual Momentum actually fails to perform better than Single Momentum in numerous categories. To be fair, Dual Momentum performed respectably in its favored SMA-125-day and SMA-250-day categories.

While the use of randomly selected funds in test strategies may sound excessively harsh, the possible tainting of conclusions with inadvertent, subtle experimental hindsight selection bias must not be underestimated. Furthermore, to be complete, the possibility of hindsight selection bias in algorithm design must also be addressed. If indeed the 12-month SMA is the ideal trend measurement algorithm, an adaptive design will either reach the same conclusion, find a better solution or fail to converge on a solution and produce poor results. A further compelling reason for an adaptive design is the inherent curiosity of active investors who will always try new fund combinations, each requiring a particular algorithm and time constant for optimal performance.

Automated Polymorphic Momentum

The design objectives for Automated Polymorphic Momentum (APM) include:

1. Automatically determining the best performing momentum algorithm from among a set of momentum algorithms for an initial sample set of data.
2. Walking forward in time through new out-of-sample data using the previously determined best performing momentum algorithm.
3. Periodically repeating step #1 utilizing an updated sample set of data.
4. Fully automating the process to allow investors to focus on the higher level task of judiciously choosing candidate sets of funds that play well together.

APM, as implemented within the Strategy Evaluation Tool of Appendix D, concurrently operates 20 strategies, each differing by algorithm selection (EMA, DEMA, or TEMA) and/or differing by time constant selection (between 12 and 120 days). The impulse responses of the four EMA, eight DEMA, and eight TEMA filters plotted in Figure 17 is intended to illustrate the breadth and uniformity of their coverage. The performance of each of the 20 concurrent strategies is first evaluated on the selected "BornOn Date" and the momentum algorithm of the best performing strategy is selected for walking forward in time through the subsequent quarter year of out-of-sample data. At quarterly intervals the performance of the

20 concurrent strategies is again evaluated, with emphasis on the most recent three years, and at each such interval the momentum algorithm of the best performing strategy is selected for walking forward in time through the subsequent quarter year of data... and so on across the span of time.

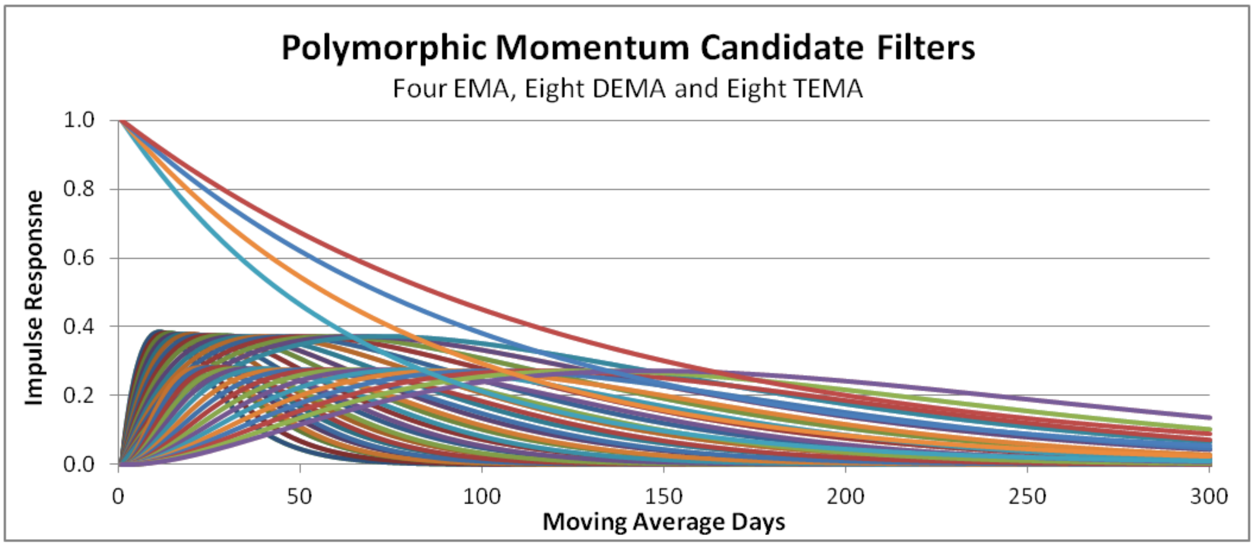


Figure 17. Impulse response plots of Polymorphic Momentum candidate filters.

The performance of APM is compared with multiple other algorithms in Figure 18a for 200 random selections of 12 funds from the Fidelity General fund set in Appendix A, in Figure 18b for 200 random selections of 12 funds from the Fidelity Sectors fund set in Appendix B, and in Figure 18c for 300 random selections of 12 ETFs from the ETFs Pre-2007 fund set in Appendix C. The values tabulated are the average value for each of the 200 (or 300) random strategies in each category. The values in the Monkey columns represent randomly selected and thus effectively equally weighted average performance of the candidate funds. The CAGR value is the average Compound Annual Growth Rate of the strategies, the Sharpe value is the average of their Sharpe Ratios, and the Max.DD value is the average of their Max Drawdowns. Notably, APM is consistently the highest performer across all three sets of data.

Fidelity General -- 200 Strategies -- 12/31/2005 to 12/31/2015

		Dual Momentum				Single Momentum				
	APM	SMA-125	SMA-250	EMA-125	DEMA-21	SMA-125	SMA-250	EMA-125	DEMA-21	Monkey
CAGR	12.0%	8.2%	8.5%	10.5%	11.4%	6.7%	7.9%	7.3%	7.7%	6.9%
Sharpe	0.65	0.36	0.47	0.53	0.57	0.24	0.33	0.28	0.30	0.27
Max.DD	20%	31%	28%	24%	23%	51%	46%	51%	53%	52%

Figure 18a. Average performance of 200 random "Fidelity General" strategies.

Fidelity Sectors -- 200 Strategies -- 12/31/2005 to 12/31/2015

		Dual Momentum				Single Momentum				
	APM	SMA-125	SMA-250	EMA-125	DEMA-21	SMA-125	SMA-250	EMA-125	DEMA-21	Monkey
CAGR	15.0%	8.4%	8.5%	12.3%	10.0%	7.7%	7.7%	10.3%	9.6%	8.2%
Sharpe	0.78	0.31	0.33	0.50	0.38	0.25	0.25	0.35	0.34	0.30
Max.DD	23%	42%	44%	39%	51%	59%	56%	58%	58%	54%

Figure 18b. Average performance of 200 random "Fidelity Sectors" strategies.

ETFs Pre-2007 -- 300 Strategies -- 12/31/2005 to 12/31/2015

		Dual Momentum				Single Momentum				
	APM	SMA-125	SMA-250	EMA-125	DEMA-21	SMA-125	SMA-250	EMA-125	DEMA-21	Monkey
CAGR	12.8%	7.7%	8.0%	8.2%	6.5%	6.7%	7.6%	7.6%	6.7%	6.2%
Sharpe	0.61	0.30	0.31	0.29	0.22	0.24	0.27	0.25	0.22	0.24
Max.DD	27%	39%	44%	44%	47%	51%	52%	53%	53%	51%

Figure 18c. Average performance of 300 random "ETF Pre-2007" strategies.

The comparative equity curves of Figure 19 provide additional insight into performance characteristic differences between APM (green), Single Momentum (red), Dual Momentum (blue), and the reference S&P 500 (black) for each of the three fund sets. For simplicity, only the SMA-250 algorithm was chosen to represent both Single Momentum and Dual Momentum in these charts. Each equity curve is the average of the full set of equity curves produced for the strategies of one of the three fund sets using the specified momentum algorithm. Noteworthy observations include, (a) the Fidelity General strategies, composed of funds that are themselves broadly diversified, responded primarily to the quality of the bear market strategy, (b) the Fidelity Sectors strategies outperformed the other categories, and (c) the presence of bond and treasury funds among the ETFs apparently helped the Single Momentum strategies (that included them) perform somewhat more like Dual Momentum strategies during bear markets. A link is provided in Appendix E to download the underlying spreadsheets for these charts.

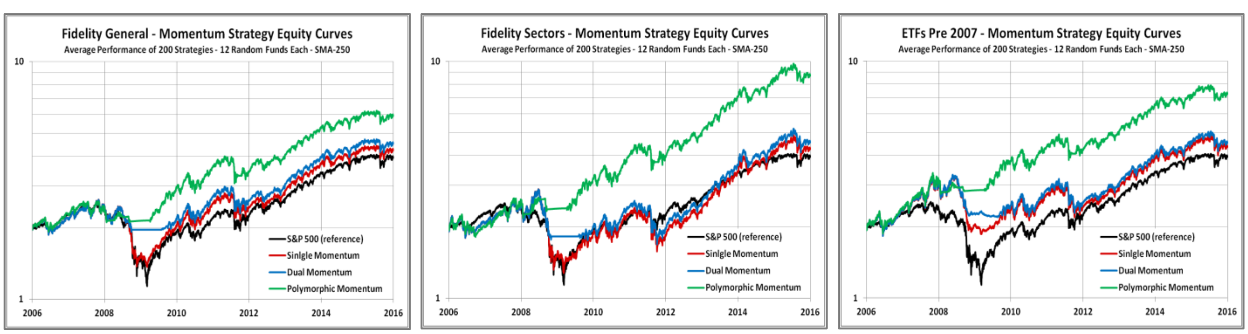


Figure 19. Averaged equity curves for each of three fund sets and four algorithms.

Conclusion

The cross-disciplinary sciences of Matched Filter Theory and differential signal processing have been shown to be transformative in their ability to improve algorithmic investment decisions through increasing the signal-to-noise ratio of a momentum algorithm's trend signal. Specifically, Matched Filter Theory improved the decision process for moving to the safety of cash during a prolonged bear market, and differential signal processing improved the probability of selecting the best performing fund next month. Even though different sets of equities require different momentum filter functions, it has been shown that a momentum filter that is adaptive in both shape and duration not only outperforms ordinary solutions, but impressively does so when subjected to the rigorous standards of random strategy construction and forward walk testing, both of which are designed to eliminate the presence of hindsight selection bias.

Appendix A

"Fidelity General" Mutual Funds List

FAGIX	Fidelity Capital & Income
FAMRX	Fidelity Asset Manager 85%
FASGX	Fidelity Asset Manager 70%
FASIX	Fidelity Asset Manager 20%
FASMX	Fidelity Asset Manager 50%
FBALX	Fidelity Balanced
FBGRX	Fidelity Blue Chip Growth
FCNTX	Fidelity Contrafund
FCVSX	Fidelity Convertible Securities
FDCAX	Fidelity Capital Appreciation
FDEGX	Fidelity Growth Strategies
FDEQX	Fidelity Disciplined Equity
FDFFX	Fidelity Independence
FDGFX	Fidelity Dividend Growth
FDGRX	Fidelity Growth Company
FDSCX	Fidelity Stock Selector SmallCap
FDSSX	Fidelity Stock Selector AllCap
FDSVX	Fidelity Growth Discovery
FDVLX	Fidelity Value
FEQIX	Fidelity Equity Income
FEQTX	Fidelity Equity Dividend Income
FEXPX	Fidelity Export & Multinational
FFFAX	Fidelity Freedom Income
FFFCX	Fidelity Freedom 2010
FFFDX	Fidelity Freedom 2020
FFFEX	Fidelity Freedom 2030
FFFFX	Fidelity Freedom 2040

FFIDX	Fidelity Fund
FFNOX	Fidelity Four In One Index Fund
FGRIX	Fidelity Growth & Income
FGRTX	Fidelity MegaCap Stock
FIUIX	Fidelity Telecom & Utilities
FIVFX	Fidelity Intn'l Capital Appreciation
FLCSX	Fidelity LargeCap Stock
FLPSX	Fidelity Low Priced Stock
FLVCX	Fidelity Leveraged Company Stock
FMAGX	Fidelity Magellan
FMCSX	Fidelity MidCap Stock
FMILX	Fidelity New Millennium
FOCPX	Fidelity OTC
FPURX	Fidelity Puritan
FRESX	Fidelity Real Estate Investment
FSCRX	Fidelity SmallCap Discovery
FSEMX	Fidelity INV:Spartan Extended Mrkt
FSLCX	Fidelity SmallCap Stock
FSLSX	Fidelity Value Strategies
FSLVX	Fidelity Stock Selector LargeCap
FSMVX	Fidelity MidCap Value
FSTMX	Fidelity INV:Spartan Total Market Ix
FTQGX	Fidelity Focused Stock
FTRNX	Fidelity Trend
FUSEX	Fidelity INV:Spartan 500 Index Fund
FVDFX	Fidelity Value Discovery

Appendix B

“Fidelity Sectors” Mutual Funds List

FSAIX	Fidelity Sel Air Transportation
FSAVX	Fidelity Sel Automotive
FSRBX	Fidelity Sel Banking
FBIOX	Fidelity Sel Biotechnology
FSLBX	Fidelity Sel Brokerage & Investment Mgmt
FSCHX	Fidelity Sel Chemicals
FSDCX	Fidelity Sel Communication Equipment
FDCPX	Fidelity Sel Computers
FSHOX	Fidelity Sel Construction & Housing
FSCPX	Fidelity Sel Consumer Discretionary
FSVLX	Fidelity Sel Consumer Finance
FDFAX	Fidelity Sel Consumer Staples
FSDAX	Fidelity Sel Defense & Aerospace
FSELX	Fidelity Sel Electronics
FSENX	Fidelity Sel Energy
FSESX	Fidelity Sel Energy Service
FSLEX	Fidelity Sel Environmnt & Altrntve Energ
FIDSX	Fidelity Sel Financial Services
FSPHX	Fidelity Sel Health Care
FSCGX	Fidelity Sel Industrial Equipment
FCYIX	Fidelity Sel Industrials
FBSOX	Fidelity Sel Information Tech Services
FSPCX	Fidelity Sel Insurance
FDLSX	Fidelity Sel Leisure
FSDPX	Fidelity Sel Materials
FSHCX	Fidelity Sel Medical Delivery
FSMEX	Fidelity Sel Medical Equipment & Systems
FBMPX	Fidelity Sel Multimedia
FSNGX	Fidelity Sel Natural Gas
FNARX	Fidelity Sel Natural Resources
FSRPX	Fidelity Sel Retailing
FSCSX	Fidelity Sel Software & Computers
FSPTX	Fidelity Sel Technology
FSTCX	Fidelity Sel Telecommunications
FSRFX	Fidelity Sel Transport
FSUTX	Fidelity Sel Utilities Growth
FWRLX	Fidelity Sel Wireless
FPHAX	Fidelity Sel Pharmaceuticals

Appendix C

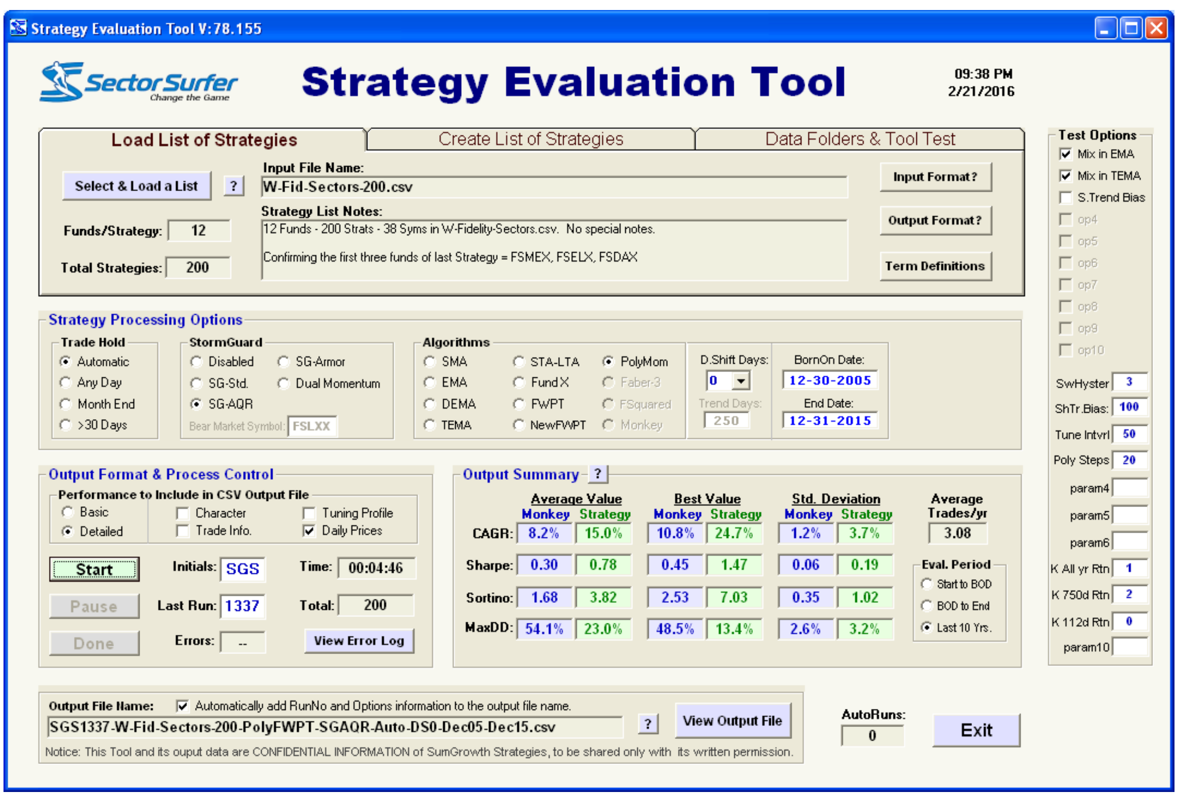
“ETFs Pre-2007” Funds List

ADRA	DVY	EZA	IGE	IXC	KRE	PRFZ	RSCO	VB	XLK
ADRD	DWAQ	EZU	IGM	IXG	KXI	PRN&	RSP	VBK	XLP
ADRE	DWM	FBT	IGN	IXJ	LQD	PSI	RTH	VBR	XLU
ADRU	DXJ	FDL	IGV	IXN	MDY	PSJ	RTM	VCR	XLV
AGG	EEB	FDM	IHE	IXP	MDYG	PSL	RWR	VDC	XLY
AUSE	EEM	FDN	IHF	IYC	MDYV	PSP	RWX	VDE	XME
AXJL	EFA	FEU	IHI	IYE	MTK	PSQ	RXI	VFH	XOP
BBH	EFG	FEZ	IJH	IYF	MXI	PTF	RYE	VGK	XPH
CCXE	EFV	FPX	IJJ	IYG	MYY	PTH	RYF	VGT	XRT
CSD	EPP	FTCS	IJK	IYH	NFO	PUI	RYH	VHT	XSD
CVY	EQWL	FVD	IJR	IYJ	OEF	PUW	RYJ	VIG	
DBC	EQWM	FVL	IJS	IYK	OIH	PWB	RYT	VIS	
DBU	EQWS	FXA	IJT	IYM	OIL	PWC	RYU	VNQ	
DBV	EVX	FXB	ILF	IYR	ONEK	PWV	RZG	VO	
DEF	EWA	FXC	INP	IYT	ONEQ	PXE	RZV	VOE	
DES	EWC	FXE	IOO	IYW	PBE	PXI	SDY	VOT	
DEW	EWD	FXF	ITA	IYY	PBJ	PXJ	SH	VOX	
DFE	EWG	FXI	ITB	IYZ	PBS	PXMG	SHY	VPL	
DFJ	EWH	FXS	ITOT	JKD	PBW	PXMV	SLV	VPU	
DGT	EWI	GDX	IUSG	JKE	PEJ	PXQ	SLX	VTI	
DHS	EWJ	GLD	IUSV	JKF	PEY	PXSG	SLY	VTV	
DIA	EWK	GNAT	IVE	JKG	PEZ	PXSV	SLYG	VUG	
DIM	EWL	GSG	IVV	JKH	PFI	PYZ	SLYV	VV	
DJP	EWM	GSP	IVW	JKI	PFM	PZD	SMH	VWO	
DLN	EWN	IAI	IWB	JKJ	PGF	PZI	SOXX	VXF	
DLS	EWO	IAK	IWC	JKK	PGJ	QQEW	SPHQ	VYM	
DNL	EWP	IAT	IWD	JKL	PHO	QQQ	SPY	WMCR	
DOG	EWQ	IAU	IWF	JPP	PID	QTEC	SPYG	XBI	
DOL	EWS	IBB	IWM	JPXN	PJP	RCD	SPYV	XES	
DON	EWT	ICF	IWN	JSC	PKB	RFG	THRK	XHB	
DOO	EWU	IDU	IWO	JXI	PKW	RFV	TIP	XLB	
DSI	EWW	IEF	IWP	KBE	PMR	RGI	TLT	XLE	
DTD	EWY	IEO	IWR	KCE	PPA	RHS	TUSA	XLF	
DTH	EWZ	IEV	IWS	KIE	PPH	RPG	USO	XLG	
DTN	EXI	IEZ	IWV	KLD	PRF	RPV	VAW	XLI	

Appendix D

Strategy Evaluation Tool

The Strategy Evaluation Tool was developed to provide a means of removing hindsight selection bias from the design of a trend following strategy's set of candidate funds, so that the merits of different algorithms could be more readily assessed. The steps for use include: (1) Create or select a universe of funds to be included (such as all Fidelity sector funds). (2) Create a set of N strategies for analysis, each with up to 12 randomly selected funds. (3) Select the Trade Hold rule, bear market strategy method, and trend algorithm. (4) Select the trend time constant (if not automated). (5) Select the "forward walk" time period for comparative analysis. (6) Click the Start button to assess the algorithm's merits.



Appendix E

Notes and References

[1] Jegadeesh, Narasimhan, and Sheridan Titman, 1993, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance* 48, 65-91

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Technical References:

[t1] Information Theory: https://en.wikipedia.org/wiki/Information_theory
[t2] Detection Theory: https://en.wikipedia.org/wiki/Detection_theory
[t3] Signal-to-noise ratio: https://en.wikipedia.org/wiki/Signal-to-noise_ratio
[t4] Matched Filter Theory: https://en.wikipedia.org/wiki/Matched_filter
[t5] Differential signal: https://en.wikipedia.org/wiki/Differential_signaling
[t6] Common mode noise https://en.wikipedia.org/wiki/Common-mode_signal
[t7] Impulse response: https://en.wikipedia.org/wiki/Impulse_response

Figure 19 Data Reference:

Spreadsheet links for Figure 19 containing full statistical and daily performance data.
Fidelity General http://SumGrowth.com/infopages/videos/APM-Fidelity-General.xlsx (33 MB)
Fidelity Sectors http://SumGrowth.com/infopages/videos/APM-Fidelity-Sectors.xlsx (35 MB)
ETFs Pre-2007 http://SumGrowth.com/infopages/videos/APM-ETFs-Pre-2007.xlsx (38 MB)